



Employee Ownership and Firm Growth

Harvard College Economics Labs

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1 Introduction

1.1 Ownership Works

Ownership Works is a non profit, that works with private firms to establish larger degrees of employee ownership, in an effort to "catalyze economic opportunity" for workers, especially those from marginalized groups. Since 04/2022, has partnered with 20 investment agencies to then work with 50 companies, a total of 65,000 employees. Their work so far has generated 153 million dollars of wealth shared with workers of color, and a total of 359 million dollars of wealth shared.

This paper explores employee ownership as a method of incentivizing worker effort and quantifies the impacts on key performance indicators like productivity and overall employee engagement. As part of this paper, a Microeconomic Model was developed to generally categorize the specific benefit and pay-off matrices of each worker and firm. Using the data provided by Ownership Works, a few statistical processes were used to analyze the data, but given the limited nature of the data-set more meaningful interpretations will require more data.

2 Team and Project Overview

2.1 Harvard College Economics Labs

Harvard College Economics Labs is a group of undergraduate researchers from Harvard that completes pro-bono economics research for governments and other social enterprises. The organization is advised by Harvard graduate researchers and faculty members. Its past and current research partners include the United Nations Development Programme (UNDP), the Virginia Department of Education, the climate think tank E3G, and Chelsea Public Schools in Chelsea, MA.

This project is being conducted by the Harvard College Economics Labs student group at Stanford University.

2.2 Proposal Proper: 3 Items

In this project we will tackle three key points:

1. Literature Review: What are the key differences in the models of employee ownership, such as direct vs. owning shares in a trust (as in an ESOT), and how do they impact firms across level of employee ownership and firm task homogeneity?
2. Micro Theory (short): Can we create a model based on previous literature, perhaps in a certain industry such as manufacturing, that explains marginal changes to productivity through incentive changes:
 - (a) How much incentive is provided by employee ownership? I.e. change incentive structure.
 - (b) How does an increase in incentive map to productivity, given a specific industry.

3. Research Design for Future Data Collection: What metrics or tests can we set up to best analyze the interaction between employee knowledge of key priorities for firm success, belief in firm success translating to individual financial interest (incentive), and productivity measures (via regression discontinuity):
- (a) Explain necessary data collection for Ownership Works. Time series selection, what subsection of employees should Ownership Works care about, how often should they collect data, sample size (power analysis if possible).
 - (b) Second, write up code to test basic sample data Ownership Works provides to us (note: they cannot provide full data so must resample from empirical $\hat{F}_n$ to create dummy samples). Answer questions like: What do we have to do when the data has XYZ shape?

3 Literature Review

3.1 Incentives and Worker Effort

Conventional intuition in behavioral economics suggests that implementing differing ownership structures could significantly impact worker incentives in a firm. However, contemporary literature demonstrates that these impacts may be mixed. Residual loss is greater in participatory firms when more people share in residual gains; thus, each individual is less willing to contribute effort to reap these gains. Participatory firms also tend to lack strong centralized accountability and require substantial resources for implementing an ownership structure. Conversely, residual loss is smaller in participatory firms when workers feel greater identification with shared productivity goals due to extra-economic motivations such as solidarity. Participatory firms also face lower monitoring costs and have high retention rates.

Many thoroughly controlled economic estimates of the effect of participation on productivity efficiency appear in a seminal paper by Michael A. Conte and Jan Svejnar (1987)¹. Conte and Svejnar compare results from OLS and instrumental variables across different participation structures to guard against covariance among participation variables. Their data indicated that profit-sharing increases productivity by approximately 32% and ESOP increases productivity by about 9%, while non-managerial direct employee ownership diminishes productivity by 17%. Conte and Svejnar ultimately postulate that moderate levels of indirect employee ownership positively impact a firm's productivity while significant amounts of direct employee ownership can harm efficiency. Because these results affirm the possibility of a non-zero optimal degree of worker participation, our model assumes a non-linear production function against ownership. The paper also highlights concerns about self-selection bias, life-cycle effects, industry discrepancies, and unrepresentative performance variables. Our model accounts for these factors by not assuming the endogeneity of participatory structures.

3.2 Impact on Employees

There has been some literature dedicated to empirically examining the effect of employee ownership models on productivity. For example, [Kim and Patel \(2020\)](#) found using corporate panel data from firms in South Korea that employee ownership alone did not significantly contribute to firm productivity. However, employee ownership was effective at improving firm productivity when both industry growth and instability are high, suggesting an interaction between industry trends and employee ownership outcomes. In a similar vein, [Pendleton and Robinson \(2010\)](#) used an interaction-based approach to measure the effect of employee stock ownership and involvement on productivity. They used data from the British Workplace Employment Relations Survey (2004)

¹Conte, M. A., & Svejnar, J. (1988). Productivity effects of worker participation in management, profit-sharing, worker ownership of assets and unionization in U.S. firms. *International Journal of Industrial Organization*, 6(1), 139–151. [https://doi.org/10.1016/0167-7187\(88\)90011-2](https://doi.org/10.1016/0167-7187(88)90011-2)

and conducted interviews with senior managers to measure the effect of employee ownership and involvement on labor productivity. Their results found that stock plans seem to need other forms of employee involvement to be effective, especially when there is minority participation in the plan. However, majority participation in ESO plans did have an independent effect on productivity.

Blasi and Kruse (2010) analyzes the relationship of various forms of shared capitalist compensation to six workplace outcomes—turnover, absenteeism, perceived effort of co-workers, loyalty to the firm, willingness to work hard, and frequency of worker suggestions to improve productivity. Blasi and Kruse (2010) use datasets from the 2002 and 2006 General Social Surveys (GSS), which cover 2430 persons in for-profit firms from a nationally representative sample; and an NBER-sponsored firm-based survey of 41,206 employees at 320 worksites in 14 companies for whom profit/gain sharing. It found that shared capitalism is linked to lower turnover and greater loyalty and willingness to work hard, particularly when combined with high-performance policies, low levels of supervision, and fixed pay at or above market levels. The only outcome with which shared capitalist compensation is adversely related is absenteeism, but this result largely disappears when controlling for interactions with high-performance policies and closeness of supervision. Most workers report that cash incentives, stock options, ESOP stock, and ESPP participation motivate them to work harder.

4 Research Design

4.1 Working with Given Data - Initial Survey Design

Ownership Works provided us with data from one firm where employees were surveyed on twelve questions and asked to rate their agreement on a scale from 1 to 5. The aggregate data from before and after the implementation of employee ownership policies was taken, and we have merged it into one data set to determine whether employee ownership impacts productivity and employee satisfaction.

To start, we assume that the number of respondents (n) is the same for both samples. The surveys, Gallup Q12 questionnaires, were sent out to all employees of the surveyed companies during the two trials before and after employee ownership was implemented, so the number of respondents should be generally the same.

$$n_1 \approx n_2$$

This allows us to calculate the degrees of freedom for the sample, which is the sum of sample sizes and subtracting 2 for error.

$$df = n_1 + n_2 - 2$$

Combs and Oltman (1996) provides guidelines for controlling Type 1 errors when population variances are not equal or indeterminable, as in this case. Based on their framework, we are also able to assume that the standard deviations (s) of the samples are equal. We are able to do this because

	Q12 Questions	Respondents (n)	Average (μ_1)	Past Average (μ_2)
0	How satisfied are you with your company as a place to work?	1769	3.84	3.69
1	I know what is expected of me at work.	1780	4.36	4.29
2	I have the materials and equipment I need to do my work right.	1776	3.89	3.78
3	At work, I have the opportunity to do what I do best every day.	1766	3.98	3.82
4	In the last seven days, I have received recognition or praise for doing good work.	1759	3.76	3.41
5	My supervisor, or someone at work, seems to care about me as a person.	1762	4.17	4.04
6	There is someone at work who encourages my development.	1762	3.98	3.76
7	At work, my opinions seem to count.	1764	3.56	3.32
8	The mission or purpose of my company makes me feel my job is important.	1767	3.77	3.64
9	My coworkers are committed to doing quality work.	1771	3.86	3.67
10	I have a best friend at work.	1660	3.30	3.09
11	In the last six months, someone at work has talked to me about my progress.	1718	3.94	3.86
12	This last year, I have had opportunities at work to learn and grow.	1710	4.02	3.87

we take as given that we and Ownership Works in the future will be comparing firms that have sufficiently large equal-size samples.

$$s_1 = s_2$$

Based on these assumptions, we are able to create a hypothesis test comparing the sample means for the past average and current average numerical ratings for each question and determine whether there is significance between the two surveys. We conduct this with a significance level of 5% and t-stat statistics as we are comparing samples and not populations.

With insignificant standard deviations and variations, we can use a common s-squared pooled method of combining the standard deviations. s-squared pooled takes in the number of respondents which was predetermined as the same for both sample 1 and sample 2, a standard deviation that is common for both samples, and the degrees of freedom found previously.

$$s_{\text{pooled}}^2 = \frac{(n_1 - 1)s_1^2 + (n_2 - 1)s_2^2}{df}$$

The test-statistic takes in the mean score given by respondents for each question and subtracts the means for the hypothesis test, which should be zero. In the denominator, the s-squared pooled value is multiplied by the inverse sum of the two sample sizes.

$$\text{t-stat} = \frac{(x_2 - x_1) - (\mu_2 - \mu_1)}{\sqrt{s_{\text{pooled}}^2 \left(\frac{1}{n_1} + \frac{1}{n_2} \right)}}$$

Finally, we calculate the critical value using the significance level of 5%, which translates to 0.05, and degrees of freedom, which is once again the sum of the two sample sizes subtracting 2.

$$\text{critical value} = \text{TInv}(0.05, df)$$

By comparing the critical value to the test-statistic, we can determine if there is any significance in the survey data pre- and post-employee ownership in the company. If the test-statistic is less than the critical value, there is no significance; if the test-statistic is greater than the critical value, the two samples are statistically significant and we can conclude that employee ownership changed the

viewpoints of employees on their place in their firms.

From our data calculations, we determine that there is no significant difference between employee satisfaction before and after the employee ownership model was implemented. One way we can determine significance by increasing the sample size to allow for larger differences in sample means and standard deviations. Another option is to increase the spread of scoring from 1-5 to 1-10, as that would increase the variance of responses and possibly show significance. A final method would be to increase treatment effect size through fighting selection bias as will be discussed in the next section.

4.2 Simulating Data and Newer Designs (Work in Progress)

There are several limitations to the data that we recognize as challenges in our analysis. First, we do not have much data for most firms prior to the intervention by Ownership Works. The lack of pre-intervention data complicates our ability to have a baseline to compare the post-intervention data. In addition, the firms that agree to work with Ownership Works are likely not representative of all firms. This selection bias suggests that our results based on the sample are not generalizable.

We would want pre-intervention data for a firm that uses the same questions asked and metrics measured as the post-intervention data. An alternative is to match data from Gallup polls. For applicable companies, we can match their Gallup data prior to the intervention to the data that Ownership Works collects after the intervention. In terms of selection bias, we can identify firms that have similar characteristics as Ownership Works' firms. Then, we can compare Gallup data on those firms with the data on Ownership Work's firms to get a better understanding of whether Ownership Work's intervention had an effect.

In order to make sense of the impact at scale, a more extensive array of data is required. In this circumstance, where the dataset does not possess large-scale volume, we plan to simulate supplemental data predicated on the statistical attributes of the existing data set. By utilizing the kurtosis, skewness, and mean of the original data, which provide a comprehensive understanding of the data's distribution, we can generate synthetic data points. To do so, we can introduce slight random variation and generate new data points that align with the original distribution, but are distinct and behave as new data points.

Further to this, our method integrates a bootstrap method to account for inherent uncertainties in our estimates. We plan to utilize bootstrap re-sampling for the generation of a second array of distinct data sets by repeatedly sampling from the original data set with replacement. Each of these new data sets can then be utilized to estimate statistical measures and propagate additional data points. This process creates new data points, representing a broad range of potential outcomes and introducing a greater degree of variability than the original data set alone could provide.

4.3 Key Metrics

These are a variety of metrics in which we can observe trends of the direct and indirect consequences of ownership structures. Notably, the OW survey data provides an explicit framework for quantitatively understanding the opinions of workers participating in ownership programs. Using a variety of techniques such as regression discontinuity and hypothesis testing, we can determine statistical correlations between ownership incentives and worker output. Additionally, indicators of participation (including voting records and rights) could be used as a proxy for differentiating ownership models and refining correlation estimates.

As such, we could study:

- Regression Discontinuity via OW survey question 4 (for given KPI): The Regression Discontinuity approach examines the effect of a specific survey question on key performance indicators. Responses to the question are used as a threshold to separate individuals into two groups. Within these groups, the causal effect of the survey question can be studied.
- "Psychological Ownership" as measured by voting rights as a proxy for employee ownership decreased costs and increased organizational productivity.²
- Interaction of "my firm succeeding is in my financial interest" and knowledge of key priorities for firm success to measure engagement, via OW survey questions 3,4, and 7:
- Hypothesis Testing of Means of Job Satisfaction:
- Longitudinal analysis of ownership metrics assuming endogeneity

More data could be collected for future investigations exploring the correlation between these metrics and worker outputs. To make our results generalizable, survey data would ideally include pre-intervention and post-intervention studies. Additionally, increasing the scoring spread and sample size of data could provide more nuanced results that reveal statistical significance by our methods.

5 Micro Model

5.1 Model

A firm employs n workers. Each worker chooses some level of effort $e \in [0, \bar{e}]$ to exert,³ which costs them $c(e)$. Normalize $c(0) = 0$. We assume that each worker faces symmetric effort costs and that $c : \mathbb{R} \rightarrow \mathbb{R}$ is strictly increasing, twice differentiable, and strictly convex: $c'(e) > 0, c''(e) > 0$. Let e_i denote worker i 's effort, where $i \in \{1, 2, \dots, n\}$.

²Impact of Employee Ownership on an Organizational Productivity: A Mediating Role of Psychological Ownership, Javed.

³Alternatively, we can allow $e \in \mathbb{R}^+$ and assume c to be unreasonably high after $\bar{e}$.

The firm has some production function $F : \mathbb{R} \rightarrow \mathbb{R}$ that maps total effort $E = \sum_i e_i$ to some level of production $F(E)$. Normalize $F(0) = 0$. We assume that F is strictly increasing, twice differentiable, and strictly concave: $F'(E) > 0, F''(E) < 0$. Without loss of generality, suppose F directly maps effort into revenue. The firm pays worker i a wage of $w(e_i)$.

The timing of the game is as follows:

1. The firm chooses wage structure w ;
2. Each agent observes the wage structure;
3. All agents simultaneously choose their level of effort to exert;
4. Payoffs are collected according to:
 - Worker i 's payoff is their wage minus their cost of effort:

$$w(e_i) - c(e_i);$$

- The firm's payoff is their profit:

$$F(E) - \sum_i w(e_i).$$

5.2 Full Information Baseline

Suppose that the firm can perfectly observe worker effort and perfectly knows the behavior of F . The firm seeks to maximize

$$F\left(\sum_i e_i\right) - \sum_i w(e_i)$$

subject to the constraint that for all i ,

$$e_i = \arg \max_{e'} w(e') - c(e').$$

To perfectly induce effort e_i^* , the firm can use the cutoff wage:

$$w(e_i) = \mathbb{I}_{e_i \geq e_i^*} c(e_i^*)$$

where $\mathbb{I}_X$ is the indicator function taking a value of 1 over the set X and 0 otherwise. Using this, we can encompass the constraint in the firm's constrained maximization problem to get that the firm's optimization problem is now to choose $\{e_i\}_{i=1}^n$ to maximize

$$F\left(\sum_i e_i\right) - \sum_i c(e_i).$$

By concavity of c , if $e_i \neq e_j$ then setting $e'_i = e'_j = \frac{e_i + e_j}{2}$ achieves the same total level of effort but at a lesser cost, so we can without loss restrict consideration to symmetric solutions of $e_i = e$ for

all i . Making this substitution gives the firm's optimization problem to choose e to maximize

$$F(ne) - nc(e).$$

A unique maximizer exists as

$$\frac{d^2}{de^2}[F(ne) - nc(e)] = n^2 F''(ne) - nc''(e) < 0$$

since $n > 0, F'' < 0, c'' > 0$. It is either characterized by the first order condition of

$$F'(ne) = c'(e)$$

or a corner solution at $e = \bar{e}$. Let e^* be the maximizer; the firm sets wages to induce this level of output from all workers. Note that in this case:

1. The socially efficient level of effort is induced, as $F'(ne) = c'(e)$ at optima implies that the marginal product is equal to the marginal cost;
2. Worker payoffs are zero, as by construction they are paid wage $w(e) = c(e)$;
3. Ownership cannot be a strict Pareto improvement, as the firm is already extracting all possible surplus so any additional welfare to workers means less profits to the firm.

5.3 Realistic Case and Ownership

In the analysis of the preceding section, three strong assumptions were made:

1. The wage-setter perfectly knows the firm's production function;
2. The wage-setter perfectly knows each worker's cost function;
3. The wage-setter can perfectly observe each worker's level of effort.

In this section, we relax these assumptions to be:

1. Only the workers themselves know the firm's production function, and the wage-setting only knows production at the current level of effort;
2. Each worker only knows their own cost function, and the wage-setter does not;
3. There exists some effort $\hat{e}$ such that the wage-setter can only tell if individual effort is above or below $\hat{e}$ (it is assumed that $\hat{e} < \bar{e}$ to avoid trivial cases).

By the second assumption, the only possible wage structures are of the form

$$a + \mathbb{I}_{e_i \geq e_i^*} b$$

where a and b cannot depend on c or e_i . Since firms are profit-maximizing, we can without loss set $a = 0$, so suppose there is some firm that starts with the wage structure

$$w(e_i) = \mathbb{I}_{e_i \geq \hat{e}} W.$$

Assume that W satisfies $W \geq c(\hat{e})$ so each worker's best response to the above wage structure is to play $e_i = \hat{e}$. Furthermore, suppose the firm has been operating with this wage structure, so $F(n\hat{e})$ is known. Going forward, let this be known as the “usual wage structure”.

Next, consider the new wage structure

$$w_\lambda(e_i) = \mathbb{I}_{e_i \geq \hat{e}} \left[\lambda \cdot F\left(\sum_i e_i\right) - \lambda \cdot F(n\hat{e}) + W \right]$$

where $\lambda \in (0, 1/n)$ and W is the same as what it is under the usual wage structure. Intuitively, this wage structure gives each worker a λ share in the firm and deducts the value of the λ share at current production from their original wage. Going forward, let this be known as the “ownership wage structure”.

Our first main result analyzes how workers will behave in response to a transition from the usual wage structure to the ownership wage structure.

Theorem 1 (Smooth Transition). *Suppose all workers exert effort level $\hat{e}$ while faced with the ownership wage structure. If worker i has a profitable unilateral deviation from exerting effort $e_i = \hat{e}$ to effort $e_i = e'$, then this deviation is a Pareto improvement (for both workers and the firm).*

In the long term, we also want to characterize equilibrium when workers are faced with the ownership wage structure. Luckily, we can get both existence and welfare results.

Theorem 2 (Characterization of Equilibrium). *Under the ownership wage structure, there exists some equilibrium efforts $\{e_i\}_{i=1}^n$ such that:*

1. $e_i \geq \hat{e}$ for all i ;
2. *Payoffs in this equilibrium are a Pareto improvement (for both workers and the firm) over the equilibrium under the usual wage structure.*

Proofs are omitted and can be found in the appendix. Instead, we will present a discussion and interpretation of these results. Both theorems suggest that everyone involved will be benefited, which has practical applications in convincing a Board of Directors to vote for a switch to the ownership wage structure. Existence of equilibrium where workers still exert effort under the ownership wage structure is non-trivial. In particular, a danger of the ownership wage schedule is that there might now be an equilibrium where no worker exerts effort.

6 The Optimal λ

6.1 Existence

Note that if $\lambda = 0$, then nothing has changed, and if $\lambda = 1/n$ then all additional revenue from changing worker incentives is split among workers, and firm profits still remain unchanged. To even ask what the optimal portion of revenue to give to workers is, an important first step is to ask whether or not an optimal split exists. A major difficulty is that there might be a multitude of equilibria. We start by ruling this case out.

Theorem 3 (Equilibrium Uniqueness). *For any $\lambda \in [0, 1/n]$, the effort-choosing game induced by an ownership wage structure with parameter λ has a unique Nash equilibrium of worker efforts.*

Proof. Towards a contradiction, suppose there are two equilibria, $\{e_i\}, \{e'_i\}$. Without loss of generality, suppose $E = \sum e_i \geq E' = \sum e'_i$. Then, there must exist some j such that $e_j > e'_j$. Since $e_j > e'_j \geq \hat{e}$, it must be that e_j is an interior solution to worker j 's payoff function. As such, $\lambda F'(\sum_i e_i) = c'(e_j)$. By the curvature of the cost and production functions, we get that

$$\lambda F' \left(\sum_i e'_i \right) \geq \lambda F' \left(\sum_i e_i \right) = c'(e_j) > c'(e'_j)$$

so

$$\lambda F' \left(\sum_i e'_i \right) > c'(e'_j)$$

and worker j has a profitable deviation to increase effort exerted. As such, $\{e'_i\}$ cannot be an equilibrium as desired. $\square$

Equilibrium uniqueness then allows us to express the Nash equilibrium of the induced game as a *function* of λ opposed to a *correspondence*. As such, it becomes simpler to show existence of a profit-maximizing λ for the firm.

Theorem 4. *Let $\Sigma(\lambda)$ denote equilibrium worker efforts if the firm implements ownership wage structure λ , and let $\pi(\lambda)$ be the firm's profit under ownership wage structure λ and workers exerting effort as described by $\Sigma(\lambda)$. Then, $\pi(\lambda)$ is continuous when the first-order condition holds, and in particular, attains a maximum on $[0, 1/n]$.*

The proof of Theorem 4 also derives some comparative statics results on $e^*(\lambda)$ and $\pi(\lambda)$. In general, concavity of π depends on the *third* derivatives of F and c , which have unknown behavior (and there are no good justifications, either from the literature or in the real world for making additional assumptions).

A corollary of this characterizes when an ownership wage structure will yield benefits.

Theorem 5. *The following are equivalent:*

1. *There exists some λ such that $e^*(\lambda) > \hat{e}$;*

2. An optimal ownership wage structure exists and gives strictly higher payoffs to the firm;
3. $e^*(1/n) > \hat{e}$.
4. $F'(n\hat{e}) > nc'(e)$

While the firm's payoff as a function of λ is generally intractable, whether or not $e^*(1/n) > \hat{e}$ is tractable. In particular, by existence and uniqueness of equilibrium, $e^*(1/n) > \hat{e}$ if and only if all agents playing $\hat{e}$ is *not* a Nash equilibrium of the induced game when $\lambda = 1/n$. This happens if and only if

$$\frac{\partial}{\partial e} \frac{1}{n} F((n-1)\hat{e} + e) - c(e) > 0$$

at $e = \hat{e}$, or equivalently,

$$F'(n\hat{e}) > nc'(\hat{e}).$$

Proof. Equivalence of (1) and (2) follows from the computations in the proof of Theorem 4. It is immediate that (3) implies (1). Finally, (1) implies (3) as the incentive to work more grows stronger as λ increases. The above discussion gives equivalence between (3) and (4). $\square$

6.2 Extensions

1. Comparative statics on how n affects the effectiveness of ownership. Currently, no analysis was done on the optimal choice of λ . On the bright side, our results show that ownership works for a large range of λ but on the down side, there are no suggestions about how ownership should be implemented. As the number of workers grows, $1/n$ falls so the restriction that $\lambda \in (0, 1/n)$ becomes tighter.
2. Reducing the assumptions on c and F . There are several ways to do this. The first would be to only require weak convexity/concavity instead of strict; changing some strict inequalities into weak inequalities should do the trick. However, there might be some issues with multiplicity of equilibria. A more ambitious extension would be to remove assumptions about differentiability and instead take a monotone comparative statics approach to derive similar results.
3. No longer requiring symmetry. Removing the assumption that each worker has the same cost function should not be too difficult since all work done is either related to the cost of an individual worker or the total cost over all workers.
4. A non-homogeneous production function. Currently, production is only a function of total effort but in many firms different workers may be putting effort into different tasks. This would be more realistic but probably doable as long as reasoning about each worker's own marginal productivity holds.

5. Incorporating uncertainty into the model. There may be external factors that cause production to be nondeterministic. Might become intractable when the firm and workers have preferences over lotteries.
6. Reducing worker knowledge of F . It is probably not realistic to expect workers to know what the firm's production function is like, especially in larger firms. A weaker assumption might be for workers to only know the local behavior of F with respect to small changes in their effort or for workers to have beliefs about F .

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A Omitted Proofs

PROOF OF THEOREM 1

Proof. Fix some worker i . Any deviation to $e' \in [0, \hat{e})$ gives worker i no wage, so their utility is $0 - c(e') \leq 0$. However, playing $\hat{e}$ gives them a payoff of

$$[\lambda F(\hat{e} + (n-1)\hat{e}) - \lambda F(n\hat{e}) + W] - c(\hat{e}) = W - c(\hat{e})$$

which is at least zero by assumption. Thus, no deviation to $e' \in [0, \hat{e})$ can be profitable.

Now, suppose worker i has a profitable deviation to some $e' > \hat{e}$. Then, $e' + (n-1)\hat{e}$ so $F(e' + (n-1)\hat{e}) > F(n\hat{e})$. Each worker $j \neq i$'s payoff increases from

$$\lambda F(n\hat{e}) - \lambda F(n\hat{e}) + W$$

to

$$\lambda F(e' + (n-1)\hat{e}) - \lambda F(n\hat{e}) + W$$

while the firm's payoff increases from

$$F(n\hat{e}) - n[\lambda F(n\hat{e}) - \lambda F(n\hat{e}) + W] = (1 - n\lambda)F(n\hat{e}) + n\lambda F(n\hat{e}) - nW$$

to

$$F(e' + (n-1)\hat{e}) - n[\lambda F(e' + (n-1)\hat{e}) - \lambda F(n\hat{e}) + W] = (1 - n\lambda)F(e' + (n-1)\hat{e}) + n\lambda F(n\hat{e}) - nW.$$

Finally, player i 's deviation is profitable by construction, so they are better off as well. $\square$

PROOF OF THEOREM 2

Proof. The existence proof will proceed in two steps. First, we consider the restricted game where each worker is forced to choose effort $e_i \geq \hat{e}$ and show existence of a pure strategy Nash equilibrium in that game. Second, we remove the restriction and show that no profitable deviation to any of the strategies that were artificially removed by the restriction.

Define the game Γ such that:

1. The set of players is exactly the set of workers;
2. Each worker i 's strategy space is $[\hat{e}, \bar{e}]$;
3. Worker i 's payoff function is

$$g_i(e_i, e_{-i}) = \lambda F\left(e_i + \sum_{j \neq i} e_j\right) - \lambda F(n\hat{e}) + W - c(e_i)$$

as $\mathbb{I}_{e_i \geq \hat{e}} = 1$ on the entire interval $[\hat{e}, \bar{e}]$.

The game has the following properties:

1. There is a finite number of players;
2. Each player's strategy set is compact as $[\hat{e}, \bar{e}]$ is closed and bounded in $\mathbb{R}$;
3. Each player's payoff is continuous in $\{e_i\}_{i=1}^n$: F is differentiable so it is continuous and $(e_i + \sum_{j \neq i} e_j)$ is linear in $\{e_i\}_{i=1}^n$ so payoffs are the composition of continuous functions and continuous;
4. Each player's payoff is quasiconcave as

$$g_i''(e_i, e_{-i}) = \lambda F'' \left(e_i + \sum_{j \neq i} e_j \right) - c''(e_i) < 0.$$

Thus, by a result in Paul Milgrom's Lecture Notes (or a standard application of Kakutani's fixed point theorem following the process of Nash's existence proof), the restricted game has a pure strategy Nash equilibrium. Let $\{e_i^*\}_{i=1}^n$ denote this pure strategy Nash equilibrium.

Now, we move onto showing how $\{e_i^*\}_{i=1}^n$ is a Nash equilibrium in the unrestricted game. The only additional deviations that are possible in the unrestricted game that are not present in the restricted game are deviating to $e' \in [0, \hat{e})$. Payoffs for any deviation of this type are $-c(e') < 0$ so it suffice to show that

$$\lambda F \left(e_i^* + \sum_{j \neq i} e_j^* \right) - \lambda F(n\hat{e}) + W - c(e_i^*) > 0.$$

As $e_j^* \geq \hat{e}$ for all j , we have that $e_i^* + \sum_{j \neq i} e_j^* \geq n\hat{e}$ so as F is increasing, $\lambda F \left(e_i^* + \sum_{j \neq i} e_j^* \right) - \lambda F(n\hat{e}) > 0$. By construction, $W - c(e_i^*) > 0$ as well, and we have our desired result.

Next, payoffs when workers play $\{e_i^*\}$ under the ownership wage structure are a pareto improvement over what happens under the usual wage structure. Worker i has payoff

$$\lambda F \left(e_i^* + \sum_{j \neq i} e_j^* \right) - \lambda F(n\hat{e}) + W - c(e_i^*)$$

under ownership and

$$W - c(\hat{e})$$

under the usual wage structure. Thus, it suffice to show that

$$\lambda F \left(e_i^* + \sum_{j \neq i} e_j^* \right) - c(e_i^*) \geq \lambda F(n\hat{e}) + c(\hat{e}).$$

If $e_i^* = \hat{e}$, then

$$\begin{aligned}\lambda F\left(e_i^* + \sum_{j \neq i} e_j^*\right) - c(e_i^*) &= \lambda F\left(\hat{e} + \sum_{j \neq i} e_j^*\right) - c(\hat{e}) \\ &\geq \lambda F(n\hat{e}) - c(\hat{e})\end{aligned}$$

as F is increasing and $\sum_{j \neq i} e_j^* \geq n\hat{e}$. Otherwise, suppose $e_i^* > \hat{e}$. Then, worker i 's optimization problem has an interior solution, so it must be that

$$\frac{d}{de_i} \left[\lambda F\left(e_i^* + \sum_{j \neq i} e_j^*\right) - c(e_i^*) \right] = 0.$$

However, by previous reasoning about concavity of the payoff function, it must be that

$$\frac{d}{de_i} \left[\lambda F\left(e_i' + \sum_{j \neq i} e_j^*\right) - c(e_i') \right] > 0$$

for all $e_i' \in [\hat{e}, e_i^*]$. Using this, we now have that

$$\begin{aligned}\lambda F\left(e_i^* + \sum_{j \neq i} e_j^*\right) - c(e_i^*) &= \lambda F\left(\hat{e} + \sum_{j \neq i} e_j^*\right) - c(\hat{e}) + \int_{\hat{e}}^{e_i^*} \lambda F'\left(e_i' + \sum_{j \neq i} e_j^*\right) - c'(e_i') de_i' \\ &\geq \lambda F\left(\hat{e} + \sum_{j \neq i} e_j^*\right) - c(\hat{e}) \\ &\geq \lambda F(n\hat{e}) - c(\hat{e})\end{aligned}$$

so each worker is (weakly) better off. Finally, the firm is better off. Firm profits are $F(\sum_i e_i^*) - \sum_i [\lambda F(\sum_i e_i^*) - \lambda F(n\hat{e}) + W]$ under the ownership wage structure and $F(n\hat{e}) - nW$ under the usual wage structure. For the firm to be better off, it suffice to show that

$$(1 - n\lambda)F\left(\sum_i e_i^*\right) \geq (1 - n\lambda)F(n\hat{e})$$

which holds as $\lambda < 1/n$ implies $1 - n\lambda > 0$ and $\sum_i e_i^* \geq n\hat{e}$ implies $F(\sum_i e_i^*) \geq F(n\hat{e})$. This finishes the proof. $\square$

PROOF OF THEOREM 4

Proof. We start by characterizing worker efforts. Let $e_i^* : [0, 1/n] \rightarrow [0, \bar{e}]$ be what worker i plays in equilibrium in response to ownership share λ , so $\Sigma(\lambda) = \{e_i^*(\lambda)\}$. By symmetry, let $e_i^* = e^*$ for all i .⁴ Suppose that $e^*(\lambda)$ is an interior solution to each worker's utility-maximization problem so

⁴The case without symmetry follows similarly, albeit with additional computation and care for when different

that the first-order condition holds:⁵

$$\lambda F'(ne^*(\lambda)) = c'(e^*(\lambda)) \implies \lambda F'(ne^*(\lambda)) - c'(e^*(\lambda)) = 0.$$

Define the function $g(\lambda, x) : [0, 1/n] \times [0, \bar{e}] \rightarrow \mathbb{R}$ by

$$g(\lambda, x) = \lambda F'(nx) - c'(x).$$

Notice that

$$\frac{\partial g}{\partial \lambda} = F'(nx) > 0$$

and

$$\frac{\partial g}{\partial x} = \lambda F''(nx) - c''(x) < 0$$

so:

1. g is injective and hence invertible on its range. As such, if $(x, y) \in g^{-1}(0)$, every worker's first order optimality condition holds, and each worker playing y is a Nash equilibrium in response to each worker getting ownership share x .
2. The conditions of the Inverse Function Theorem hold, so there exists some smooth function e^* such that

$$g(x, y) = 0 \text{ if and only if } y = e^*(x).$$

With a slight change of notation, we get that $e^*(\lambda)$ is the effort each worker exerts in equilibrium when given ownership share λ .

Finally, we can write firm profits as

$$\begin{aligned} \pi(\lambda) &= F(ne^*(\lambda)) - nw(e^*(\lambda)) \\ &= F(ne^*(\lambda)) - n[\lambda F(ne^*(\lambda)) - \lambda F(n\hat{e}) + W] \\ &= (1 - n\lambda)F(e^*(\lambda)) + n\lambda F(n\hat{e}) - nW \end{aligned}$$

which is the composition of continuous functions, and is continuous itself.

What are the comparative statics of e^* and π ? The implicit function theorem gives that

$$\frac{\partial e^*}{\partial \lambda} = -\frac{1}{\partial g / \partial x} \Big|_{e^*(\lambda)} \cdot \frac{\partial g}{\partial \lambda} \Big|_{\lambda} = \frac{-F'(ne^*(\lambda))}{\lambda F''(ne^*(\lambda)) - c''(e^*(\lambda))}$$

which is generally positive, as expected.

Next, we can use this to calculate changes in firm profit as ownership share to workers change. We

workers switch from a corner to an interior solution.

⁵In a slight abuse of notation, $ne^*(\lambda)$ should technically be written out as $(n-1)e^*(\lambda) + e_i^*(\lambda)$. Interpret $F'(ne^*(\lambda))$ as $F'(\cdot)$ evaluated at $ne^*(\lambda)$ instead of a function of $ne^*(\lambda)$.

have

$$\begin{aligned}
\pi'(\lambda) &= -nF(ne^*(\lambda)) + (1 - n\lambda)F'(e^*(\lambda))ne^{*'}(\lambda) + nF(n\hat{e}) \\
&= -nF(ne^*(\lambda)) + (1 - n\lambda)F'(ne^*(\lambda))n \frac{-F'(ne^*(\lambda))}{\lambda F''(ne^*(\lambda)) - c''(e^*(\lambda))} + nF(n\hat{e}) \\
&= n[F(n\hat{e}) - F(ne^*(\lambda))] - \frac{n(1 - n\lambda)[F'(ne^*(\lambda))]^2}{\lambda F''(ne^*(\lambda)) - c''(e^*(\lambda))}.
\end{aligned}$$

$$\pi'(\lambda) = n[F(n\hat{e}) - F(ne^*(\lambda))] - \frac{n(1 - n\lambda)[F'(ne^*(\lambda))]^2}{\lambda F''(ne^*(\lambda)) - c''(e^*(\lambda))}.$$

Note that at the point where workers have an additional incentive to start working (the value of λ' such that $e^*(\lambda) > \hat{e}$ if and only if $\lambda > \lambda'$), we have that $e^*(\lambda') = \hat{e}$ so

$$\pi'(\lambda) = n[F(n\hat{e}) - F(ne^*(\lambda))] - \frac{n(1 - n\lambda)[F'(ne^*(\lambda))]^2}{\lambda F''(e^*(\lambda)) - c''(e^*(\lambda))} = -\frac{n(1 - n\lambda)[F'(ne^*(\lambda))]^2}{\lambda F''(e^*(\lambda)) - c''(e^*(\lambda))} > 0.$$

□