

Multi-Period CVaR-Optimal Bank Liquidity Allocation Under Fat-Tailed Risk and Regime-Conditional Scenarios (MPCOBLa-fat), via ADMM

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0.1 Introduction

US Category III banks ($\geq \$250\text{B}$) allocate assets under Basel III LCR, NSFR, and capital constraints, typically via rule-based heuristics that implicitly assume Gaussian return dynamics and so understate the probability of large joint losses. Taleb [3] argues (and in the opinion of the authors, correctly) that this understates tail loss. Rockafellar and Uryasev [1] showed CVaR admits a tractable convex reformulation in a single-period case; however, banks manage over multiple quarters under changing market conditions. Multi-period CVaR is harder due to time inconsistency [4], and existing multi-period work (on portfolios and portfolio management like [5]) uses mean-variance objectives without bank regulatory constraints or regime-conditional scenarios, focused instead more on returns.

For a loss L and confidence level β , the Value-at-Risk $\text{VaR}_\beta(L)$ is the β -quantile of L , and the Conditional Value-at-Risk $\text{CVaR}_\beta(L) = \mathbb{E}[L \mid L \geq \text{VaR}_\beta(L)]$ is the average loss in the worst $(1 - \beta)$ fraction of outcomes. CVaR is coherent and convex in the portfolio weights, whereas VaR is neither, which is what makes the formulation below tractable.

Our contributions: (i) a multi-period bank liquidity formulation with the full Basel III constraints, an initial-allocation anchor w_0 (calibrated to a US Category III bank, and a hard per-period ℓ_1 turnover budget, as a convex program; (ii) a comparison of two risk models, parametric Gaussian and a regime-tagged historical bootstrap; (iii) a regime-conditional scenario structure (4 normal $\rightarrow$ 2 recession $\rightarrow$ 4 recovery) coupled across periods by the turnover budget, which yields a realistic multi-quarter rebalancing glide; (iv) an ADMM decomposition whose turnover update is a closed-form projection onto an ℓ_1 ball.

0.2 Single-Period Formulation

Let $w \in \mathbb{R}^{10}$ be asset-class weights (in reality $\gg 10$) and $L_s(w) = -r_s^\top w$ the loss under scenario s . The Rockafellar–Uryasev identity discretized on N_s scenarios with epigraph variables $z_s \geq 0$ gives

$$\min_{w, \gamma, z} \quad \gamma + \frac{1}{N_s(1-\beta)} \sum_{s=1}^{N_s} z_s - \tau \hat{\mu}^\top w \quad \text{s.t.} \quad z_s \geq -r_s^\top w - \gamma, \quad z_s \geq 0, \quad w \in \mathcal{W}, \quad (1)$$

jointly convex in (w, γ, z) , with $\gamma^* = \text{VaR}_\beta(w^*)$ recovered as a byproduct. The feasible set $\mathcal{W}$ encodes Basel III as linear constraints on w (asset side); liabilities are fixed and calibrated from Call Report data. The Liquidity Coverage Ratio (LCR) requires High-Quality Liquid Assets (HQLA) to cover 30-day stressed net cash outflows: $(h^\top w)A \geq \text{NCO}_{30d}$, where h is the HQLA-tier haircut vector [7] (Level 1 assets count at $h = 1$, lower tiers at a discount). Level-2 / Level-2B composition caps (BCBS d238 Annex 1): $0.85w_{\text{L2A}} + 0.50w_{\text{L2B}} \leq \frac{2}{3}w_{\text{L1}}$ and $0.50w_{\text{L2B}} \leq \frac{15}{85}(w_{\text{L1}} + 0.85w_{\text{L2A}})$. NSFR: $(\phi^\top w)A \leq \text{ASF}$ [8]. Capital: $\kappa(\omega^\top w)A \leq \text{Capital}$, with $\kappa = 13\%$ the effective minimum total-capital ratio (Appendix A) [9]. Budget $\mathbf{1}^\top w = 1$, $w \geq 0$. Parameter values are in Appendix A. $\mathcal{W}$ is polyhedral and (1) is an LP.

0.3 Multi-Period Extension via ADMM

Extending (1) to T quarterly periods, anchored at w_0 , with a hard per-period turnover budget δ in place of a soft penalty (we use δ for the budget since κ already denotes the capital ratio):

$$\min_{w_1, \dots, w_T} \quad \sum_{t=1}^T [\text{CVaR}_\beta^{(t)}(w_t) - \tau \hat{\mu}_t^\top w_t] \quad \text{s.t.} \quad w_t \in \mathcal{W}_t, \quad \|w_{t+1} - w_t\|_1 \leq \delta \quad (t = 0, \dots, T-1), \quad w_0 \text{ fixed.} \quad (2)$$

$\text{CVaR}_\beta^{(t)}$ uses period-specific scenarios (regime-conditional). This is a pre-committed plan [4, 5]. The budget $\|w_{t+1} - w_t\|_1 \leq \delta$ is a no-fire-sale constraint: a bank cannot reposition an arbitrary fraction of its balance sheet in a single quarter. Unlike a soft penalty, which trades off the total amount of rebalancing but leaves the optimizer free to do it all at once, the budget bounds the rate of rebalancing and forces any required rotation to be spread across quarters. Introducing $y_t = w_{t+1} - w_t$ separates the smooth-and-constrained part from the nonsmooth turnover term, now an indicator:

$$\min_{\mathbf{w}, \mathbf{y}} \underbrace{\sum_{t=1}^T [\text{CVaR}_\beta^{(t)}(w_t) - \tau \hat{\mu}_t^\top w_t + I_{\mathcal{W}_t}(w_t)]}_{f(\mathbf{w})} + \underbrace{\sum_{t=0}^{T-1} I_{\{\|\cdot\|_1 \leq \delta\}}(y_t)}_{g(\mathbf{y})} \quad \text{s.t.} \quad y_t = w_{t+1} - w_t. \quad (3)$$

Both f, g are closed convex, so ADMM converges for any $\rho > 0$ [2]. Scaled-form updates: $\mathbf{w}^{k+1} = \arg \min_{\mathbf{w}} f(\mathbf{w}) + \frac{\rho}{2} \sum_t \|(w_{t+1} - w_t) - y_t^k + u_t^k\|_2^2$; $y_t^{k+1} = \Pi_{\{\|\cdot\|_1 \leq \delta\}}((w_{t+1}^{k+1} - w_t^{k+1}) + u_t^k)$; $u_t^{k+1} = u_t^k + (w_{t+1}^{k+1} - w_t^{k+1}) - y_t^{k+1}$. The w -update is a convex QP solved with CVXPY at each iteration. Because g is now an indicator, its proximal map is the Euclidean projection onto the ℓ_1 ball, $\Pi_{\{\|\cdot\|_1 \leq \delta\}}(v) = \text{sign}(v) \max(|v| - \theta, 0)$ with $\theta \geq 0$ chosen so that $\|\Pi(v)\|_1 = \delta$ (the $O(n \log n)$ sort-and-threshold of Duchi et al. [6] when $\|v\|_1 > \delta$, else v). We test fixed $\rho = 1$ and a simple adaptive variant that adjusts ρ when the primal and dual residuals are unbalanced.

We treat the regime path as a known, deterministic sequence and solve the full horizon jointly. We intend this to answer the planning question faced by a bank that holds a strong prior, namely, given a firm view that a recession arrives in a known number of quarters, how should the balance sheet be repositioned starting today?

0.4 Why a Budget in lieu of a Penalty

Figure 1 sweeps the soft penalty weight $\lambda \in \{0, 10^{-3}, 10^{-2}, 10^{-1}, 1\}$ on the otherwise identical problem and reports the initial rebalance $\|w_1 - w_0\|_1$ and the mean inter-period turnover. At $\lambda = 0$ the periods decouple and each jumps to its own optimum, giving a large initial rebalance ($\|w_1 - w_0\|_1 = 1.55$) and large ongoing churn (mean 0.32); as λ grows the trajectory shrinks until it freezes at w_0 (0.26 then 0.09, zero churn). At no setting does the penalty schedule turnover evenly across quarters, instead only the hard budget does. This earlier fused-lasso behavior produced static trajectory in our earlier work (and was commented on by peer reviewers), and is the main reason we adopt a hard budget of δ above in (2).

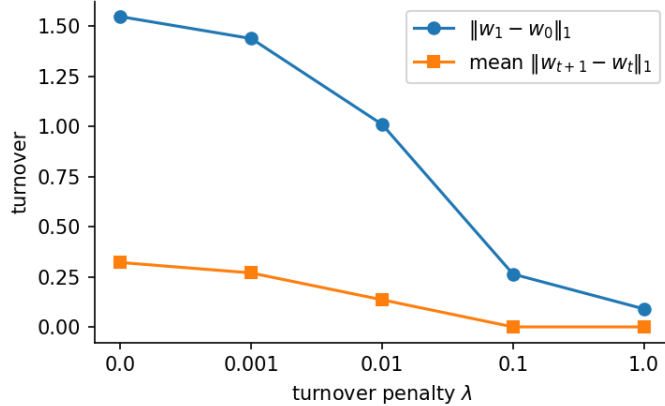


Figure 1: Soft-penalty sweep: initial rebalance and mean inter-period turnover versus λ . The penalty trades total movement for stasis but cannot spread turnover across periods, so we currently chose the aforementioned hard budget.

0.5 Data and Calibration

Ten asset classes understates the complexity of real life and banks but is diverse enough for imitating a balance sheet. 8 are market-priced (total-return indices from Bloomberg/FRED); the 2 non-market-priced classes (`ci_loan` and `whole-loan mortgage`) are constructed from FRED rate and charge-off series, enriched with a

mark-to-market component so that their volatility is realistic (about 4% annualized, comparable to the market-priced classes, rather than the sub-1% the raw carry-minus-charge-off produced; Appendix C). The muni slot uses a Vanguard tax-exempt fund basket as a proxy for the ICE BofA Muni Master (throttled by FRED) (Appendix B). Here agency MBS denotes agency mortgage-backed securities; muni denotes US municipal bonds; corp AAA and corp BAA denote corporate-bond total-return indices rated AAA and BBB respectively (BAA is Moody’s notation for the BBB tier). Basel parameters (h_i, ϕ_i, ω_i) are in Appendix A.

Asset	Source	HQLA tier	h_i	ϕ_i	ω_i
cash	FRED DGS3M0	Level 1	1.00	0.00	0.00
sov_short (1-3Y)	Bloomberg LT01TRUU	Level 1	1.00	0.05	0.00
sov_long (7-10Y)	Bloomberg LT09TRUU	Level 1	1.00	0.05	0.00
tips	Bloomberg LBUTTRUU	Level 1	1.00	0.05	0.00
agency_mbs	Bloomberg LUMSTRUU	Level 2A	0.85	0.15	0.20
muni	Vanguard proxy (App. B)	Level 2A	0.85	0.15	0.20
corp_aaa	Bloomberg BQF1TRUU	Level 2A	0.85	0.15	0.20
corp_baa	Bloomberg BCRBTRUU	Level 2B	0.50	0.50	1.00
ci_loan	constructed (App. C)	non-HQLA	0.00	0.85	1.00
mortgage (whole)	constructed (App. C)	non-HQLA	0.00	0.85	0.50

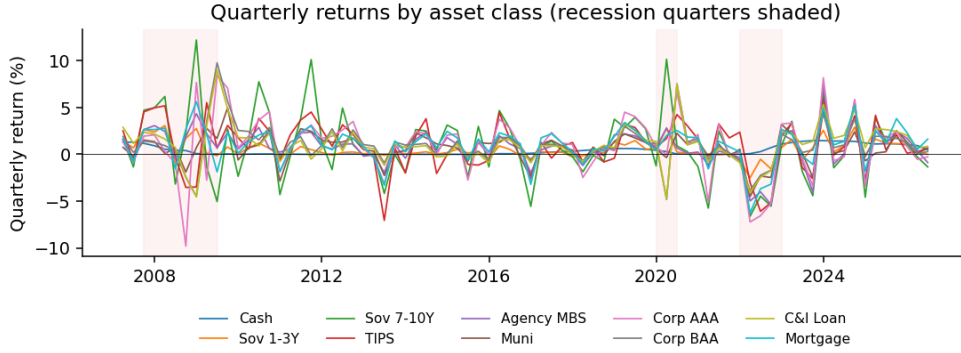


Figure 2: Quarterly total returns for the ten asset classes, 2007Q1–2026Q2.

The BBB corporate index (BCRBTRUU, starts Nov 2006) is the binding-history constraint (latest start). Final panel is 2007Q1–2026Q2, 78 quarterly observations; Figure 2 plots the ten series. Each observation is tagged normal, recession, or recovery; recession windows are GFC 2007Q4–2009Q2, COVID 2020Q1–2020Q2, and the 2022 Fed tightening 2022Q1–2022Q4. Realized counts: 49 normal, 13 recession, 16 recovery. Empirical fat-tail behaviour against a Gaussian fit (Q–Q plots, Appendix E) is visible on multiple assets, confirming doubts on Gaussian nature.

Two scenario sets: a Gaussian fit to the 49 normal quarters, and a historical bootstrap (in the single-period: iid from the full 78-quarter panel thus better representing reality; in the multi-period: regime-blocked with recession quarters sampled from real recessionary quarters, over $T = 10$). We evaluate each model’s optimal portfolio under both its own model and the empirical bootstrap (cross-evaluation) because the operative question for a bank is realized tail loss, not in-model risk.

We calibrate the bank to PNC Financial Services Group’s Q1 2026 regulatory filing (Appendix D): $A = \$482.4\text{B}$, $\text{NCO}_{30d} = \$48.3\text{B}$, $\text{ASF} = \$395.0\text{B}$, $\text{Capital} = \$49.7\text{B}$, $\kappa = 13\%$. The initial allocation w_0 is PNC’s own asset composition, $w_0 = (6.6, 8.0, 1.5, 0.0, 18.6, 0.5, 1.5, 0.0, 53.1, 10.3)\%$ (cash, sov_s, sov_l, tips, mbs, muni, aaa, baa, ci, mort), a loan-heavy book (63% loans). At w_0 the regulatory ratios are LCR 267%, NSFR 143%, and total capital 17% (against the 13% minimum), all non-binding.

0.6 Results

0.6.1 Single Period

With $\beta = 0.95$, $\tau = 1$, $N_s = 20,000$, we solve (1) under each scenario set; Table 1 reports weights and CVaR. The main finding is the cross-evaluation: the Gaussian-optimal portfolio reports $\text{CVaR}_{95} = 0.74\%$ under its own Gaussian fit but 1.36% under the empirical historical bootstrap (Figure 4), an understatement of 0.63pp. The two models also pick visibly different portfolios (Figure 3): both saturate the 30% cash cap, but under $\tau = 1$ the Gaussian optimum reaches for return, holding 25% BBB credit and 19% muni alongside 26% short sovereigns, whereas the bootstrap optimum keeps the residual in short sovereigns (62%) with only token credit, treating the fat-tailed assets with more caution.

Table 1: Single-period CVaR-optimal portfolios. Weights in %; CVaR and mean are quarterly. The Gaussian-optimal portfolio re-evaluated under the empirical bootstrap is the Taleb-point row (1.36% vs. its own-model 0.74%). More on CVaR under distributions in Figure 4.

	Gaussian	Historical
cash	30.0	30.0
sov_short	25.6	62.5
sov_long	0.0	0.0
tips	0.0	0.0
agency_mbs	0.0	0.0
muni	19.1	0.0
corp_aaa	0.0	0.0
corp_baa	25.3	1.7
ci_loan	0.0	5.8
mortgage	0.0	0.0
CVaR ₉₅ , own model	0.74%	0.92%
CVaR ₉₅ , hist. bootstrap	1.36%	0.92%
Mean qtr. return	0.63%	0.51%

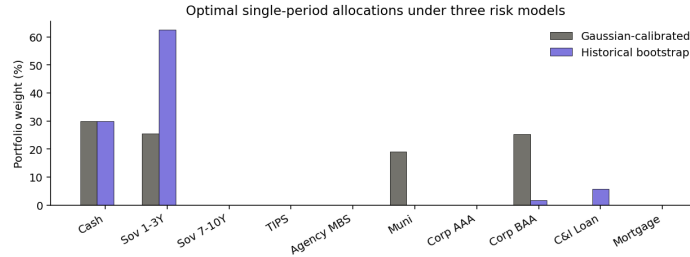


Figure 3: Optimal single-period allocations under the Gaussian and historical-bootstrap models.

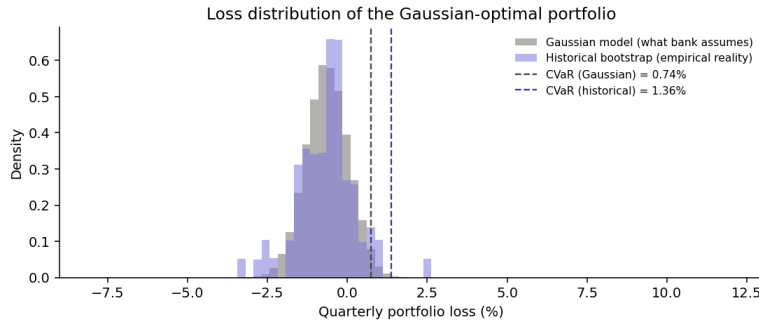


Figure 4: Loss distribution of the Gaussian-optimal portfolio under its own Gaussian fit (gray) and under the empirical historical bootstrap (blue). Dashed verticals mark realized CVaR_{95} under each: the empirical CVaR (1.36%) sits farther in the tail than the Gaussian-fit CVaR (0.74%).

0.6.2 Multi-Period with Regime Arrival in Q5

We solve (2) with $T = 10$, $N_s = 5,000$ per period, $\tau = 1$, turnover budget $\delta = 0.15$, regime normal (Q1–Q4) $\rightarrow$ recession (Q5–Q6) $\rightarrow$ recovery (Q7–Q10). We solve it two ways: directly, as one centralized LP over all ten quarters, and via the ADMM decomposition introduced above. Both return the same optimal trajectory (they agree to within 0.008 per asset per quarter), so the glide reported here is solver-independent; we use ADMM’s convergence as a check that the decomposition recovers the centralized optimum and so that it can scale for a bank’s realistic asset structure.

Under the budget the optimal plan, the initial rebalance is $\|w_1 - w_0\|_1 = .15$ and the budget binds every quarter. Starting from PNC’s loan-heavy balance sheet (53% C&I, 10% whole-loan mortgage, 19% agency MBS), ahead of and into the recession (Q1–Q6) the bank runs down its tradeable, rate-sensitive securities, the whole-loan mortgage book to zero by Q3 and agency MBS to zero by Q5, and builds cash to its 30% ceiling with short Treasuries at 22%, while the illiquid C&I book is trimmed only gradually (53% to 40%). Through the recovery (Q7–Q10) it reverses: cash is spent down from 30% to 4% and redeployed into BBB credit (0 to 15%), muni (0 to 13%), and back into C&I (40% to 48%), chasing the higher recovery-quarter returns that the $\tau = 1$ objective rewards (Table 2, Figure 5).

Some qualifications: First, because the plan has perfect foresight of a recession, de-risking begins immediately, so there is no sharp kink at the transition. This, of course, assumes perfect knowledge of a recession. Second, the early agency-MBS run-off is reinforced by the Basel Level-2 composition cap, which w_0 initially breaches, so part of the Q1–Q3 securities sell-down is constraint-driven as well as risk-driven. With four recovery quarters and the 0.15 budget the bank has room to re-risk. This quite nicely ends up modeling the reality of the full liquidity cycle a bank actually runs. Re-solving with the recession arriving in Q3, Q5, or Q7 shows that the later the downturn, the more the bank ends with more idle cash (Figure 6).

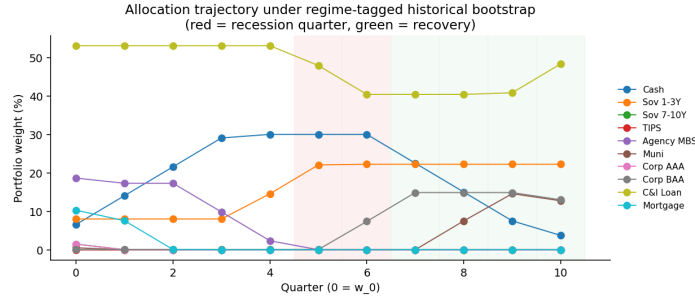


Figure 5: Optimal allocation trajectory, normal $\rightarrow$ recession $\rightarrow$ recovery, under the per-period turnover budget $\delta = 0.15$.

t	regime	cash	sov_s	sov_l	tips	mbs	muni	aaa	baa	ci	mort
w_0	init	6.6	8.0	1.5	0.0	18.6	0.5	1.5	0.0	53.1	10.3
1	normal	14.1	8.0	0.0	0.0	17.3	0.0	0.0	0.0	53.1	7.6
2	normal	21.6	8.0	0.0	0.0	17.3	0.0	0.0	0.0	53.1	0.1
3	normal	29.1	8.0	0.0	0.0	9.8	0.0	0.0	0.0	53.1	0.0
4	normal	30.0	14.6	0.0	0.0	2.3	0.0	0.0	0.0	53.1	0.0
5	recession	30.0	22.1	0.0	0.0	0.0	0.0	0.0	0.0	47.9	0.0
6	recession	30.0	22.2	0.0	0.0	0.0	0.0	0.0	7.4	40.4	0.0
7	recovery	22.5	22.2	0.0	0.0	0.0	0.0	0.0	14.9	40.4	0.0
8	recovery	15.0	22.2	0.0	0.0	0.0	7.5	0.0	14.9	40.4	0.0
9	recovery	7.5	22.2	0.0	0.0	0.0	14.6	0.0	14.9	40.8	0.0
10	recovery	3.7	22.2	0.0	0.0	0.0	12.7	0.0	13.0	48.3	0.0

Table 2: Multi-period optimal allocation, $T = 10$, starting from PNC’s w_0 . Each quarter moves at most $\delta = 0.15$ in ℓ_1 ; the budget binds every quarter.

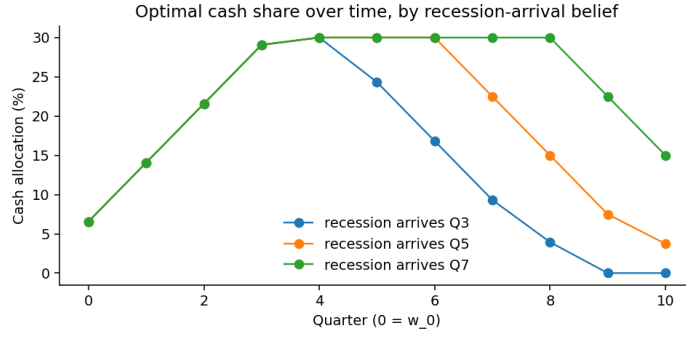


Figure 6: Optimal trajectory under earlier vs. later recession arrival (Q3, Q5, Q7).

0.6.3 ADMM Convergence

Having reported the trajectory above, we confirm the ADMM decomposition reproduces it. With $\rho = 1$ (Figure 7, Table ??), under the ℓ_1 -ball projection update it converges in 87 iterations to a maximum per-entry allocation gap of 0.008 against the centralized LP. ADMM is slower than the centralized solve at this size (each iteration re-solves a QP); its purpose is that it could be faster in a larger asset environment.

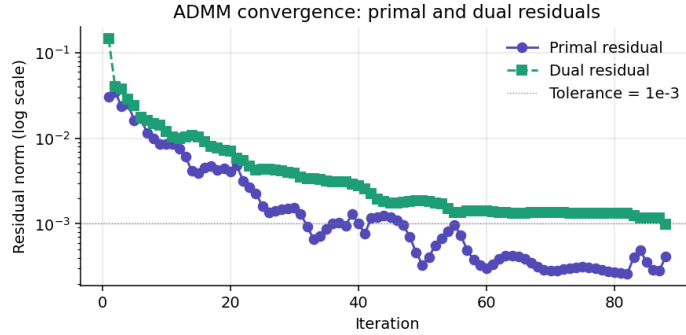


Figure 7: ADMM primal and dual residuals, fixed $\rho = 1$. Converges at iteration 87.

0.7 Conclusion

We formulate multi-period bank liquidity allocation under Basel III LCR, NSFR, and capital constraints as a mean-CVaR program with a hard per-period ℓ_1 turnover budget, and decompose it via ADMM into per-quarter QP subproblems coupled by a closed-form projection onto an ℓ_1 ball. Calibrated to PNC's Q1-2026 balance sheet, the model produces a multi-quarter rebalancing trajectory that builds cash and sheds rate-sensitive assets ahead of the recession, then redeploys into credit through the recovery. The cross-evaluation quantifies the cost of a Gaussian risk model: the Gaussian-optimal book understates its own tail loss by 0.63 percentage points out of sample. The ADMM decomposition recovers the centralized optimum to within 0.008 per asset-period, confirming the splitting is correct and viable at this scale.

Based on feedback from the teaching staff, future work would pursue two directions: (1) using an ℓ_0 (cardinality) constraint in place of the ℓ_1 norm in $\|w_{t+1} - w_t\|_1 \leq \delta$, which would limit how many assets move perquarter rather than total volume. We think this is tractable and perhaps great as the ADMM y -update just becomes a keep- K -largest hard threshold. But here ℓ_0 is nonconvex(!) so we would lose the current global convergence guarantee. (2) We could also extend to a much larger asset universe that better represents a bank's real balance sheet, where ADMM's per-quarter decomposition may be more efficient than solving the full centralized LP, and we hope this would be the case.

Finally as a thank you: we wanted to say a thank you for the class material + teaching + project scaffolding. We really enjoyed working on this problem and it's both of us' last class at Stanford and we felt it ended on a very high note of learning. Thank you!

A Basel III Parameters and Capital Buffer Decomposition

- HQLA factors h_i from BCBS d238 [7], paragraphs 49–54: Level 1 carries 0% haircut ($h = 1.00$); Level 2A (qualifying corporate bonds rated AA– or higher, 20%-RW sovereign debt, qualifying agency MBS, qualifying municipal bonds) carries 15% haircut ($h = 0.85$); Level 2B (corporate bonds A+ to BBB–) carries 50% haircut ($h = 0.50$). Composition caps (Annex 1): Level 2 $\leq 40\%$ of total HQLA post-haircut; Level 2B $\leq 15\%$ of total HQLA.
- Agency MBS as Level 2A: BCBS d238 §54(a) permits qualifying RMBS as Level 2B with a 25% haircut; US agency MBS receive Level 2A treatment per the US LCR final rule (12 CFR Part 249.20). The Bloomberg US MBS Index is constructed almost entirely from agency MBS, so Level 2A is appropriate.
- Municipal bonds as Level 2A: BCBS d238 §52(a) allows 20%-RW PSE debt at Level 2A subject to national-supervisor discretion. The Federal Reserve added qualifying US municipal securities to Level 2A in 2016.
- RSF factors ϕ_i from BCBS d295 [8]: cash and central bank reserves 0%; Level 1 HQLA 5%; Level 2A HQLA 15%; Level 2B HQLA 50%. Paragraph 42(b): “other unencumbered performing loans that do not qualify for the 35% or lower risk weight under the Basel II standardised approach for credit risk and have residual maturities of one year or more, excluding loans to financial institutions” receive 85% RSF. For both `ci_loan` (100% RW) and our 30Y whole-loan mortgage (50% RW), we use the 85% factor.
- Standardized risk weights ω_i from BCBS d424 [9] as a discussion among other BCBS documents like d374, this is a bit of a generalization: sovereigns with high credit rating 0%; agency MBS 20%; AA-rated corporates 20%; BBB-rated corporates 100%; residential mortgages 35–50% depending on LTV (we use 50% for high-LTV 30Y). For `ci_loan`, BCBS d424 §41 specifies 100% RW for exposures to corporates not rated externally; real C&I books mix rated and unrated, and we use the unrated fallback as a conservative simplification.
- Capital ratio: the model uses $\kappa = 13\%$, the effective minimum Total Capital ratio a Category III bank actually faces. The Basel III pillar-1 minimum is 8% (BCBS d189); a Cat III bank adds the following buffers, measured against CET1:
 - Pillar-1 minimum CET1: 4.5%
 - Capital Conservation Buffer: +2.5% (BCBS d189, universal)
 - Stress Capital Buffer: at least +2.5% (Federal Reserve, CCAR-determined, bank-specific)
 - Countercyclical Buffer: 0–2.5% (currently 0% in US)
- Total effective CET1 minimum $\sim 9.5\%$, Total Capital effective minimum $\sim 13\%$. We use $\kappa = 13\%$ (the $\sim 8\%$ pillar-1 floor plus the conservation and stress buffers, see appendix D for explanation) so the capital constraint reflects the binding total-capital requirement rather than the bare pillar-1 minimum.
- US bank tailoring (OCC/FRB/FDIC, 84 Fed. Reg. 59,230, Nov 2019) [10]: Category I (US GSIBs, full LCR / NSFR), II (\$700B+ or \$75B+ cross-jurisdictional, full LCR / NSFR), III ($\geq \$250B$; full LCR / NSFR if weighted short-term wholesale funding $\geq \$75B$, else 85%), IV ($\geq \$100B$ not meeting III, scaled LCR / NSFR), below \$100B (no LCR / NSFR). We model a Category III bank of course subject to full LCR / NSFR.

B Municipal Bond Proxy

The ICE BofA US Municipal Master Total Return Index (BAMLM0AOCMTRIV) is not available through any public free source with sufficient daily history. We substitute an equal-weighted total-return proxy from three Vanguard tax-exempt mutual funds spanning the muni maturity curve: VWSTX (short-term), VWITX (intermediate), VWLTX (long-term). All three are actively managed, investment-grade, nationally diversified across US states. They are moreover easy pulls with daily NAV history from Yahoo Finance using dividend-adjusted closing prices (which behave as total-return levels). The basket is built by computing each fund’s daily total return, equal-weighting across the three (NaN-aware, using whichever funds are live on a given date dropping the rest), and compounding into a level series. A nice pro of this approach is that the basket is an equal-weighted rather than market-cap-weighted aggregation across the curve. It captures the same systematic risk factors (tax-exempt rate sensitivity, muni credit) while introducing tracking error.

C Constructed Returns for `ci_loan` and `mortgage`

C&I loans and whole-loan mortgages are not market-priced, so we construct quarterly returns from FRED rate and charge-off series. A pure carry-minus-charge-off return captures interest income and realized losses but omits the change in economic value when rates or spreads move; held at amortized cost, both assets then showed implausibly low volatility (under 1% annualized) and dominated the optimizer as near risk-free carry. We therefore add the missing mark-to-market component, differently for each asset because their risks differ.

The C&I book is floating-rate (it re-prices with prime), so it carries negligible rate duration but its economic value falls when credit spreads widen. We model

$$r_t^{\text{CI}} = \frac{1}{4} \overline{\text{DPRIME}}_{[t-1]} - \frac{1}{4} \text{CORBLACBS}_t + \beta_{\text{CI}}(r_t^{\text{baa}} - \bar{r}^{\text{baa}}), \quad (4)$$

where $\bar{\cdot}_{[t-1]}$ averages over the prior quarter, `DPRIME` is the Bank Prime Loan Rate, `CORBLACBS` the C&I net charge-off rate, r^{baa} the BBB corporate total return already in the panel, and $\beta_{\text{CI}} = 1.0$ a credit-cycle sensitivity to centered BBB returns.

The whole-loan mortgage book is fixed-rate, so its fair value moves inversely with rates; we add a rate-duration mark,

$$r_t^{\text{mortgage}} = \frac{1}{4} \overline{\text{MORTGAGE30US}}_{[t-1]} - \frac{1}{4} \text{CORSFRMACBS}_t - D \Delta \text{MORTGAGE30US}_t, \quad (5)$$

where `MORTGAGE30US` is the 30Y Freddie Mac PMMS rate, `CORSFRMACBS` the single-family residential charge-off rate, $\Delta \text{MORTGAGE30US}_t$ its quarter-on-quarter change (decimal), and $D = 4.5$ years a prepayment-adjusted effective duration for this book. All series are from FRED. With this mark the mortgage book’s annualized volatility rises from about 1% to about 4%, comparable to agency MBS, and its recession-quarter behaviour reflects the 2022 rate shock that repriced fixed-rate portfolios. The optimizer no longer treats either constructed asset as free carry.

We do not add a fixed origination spread over prime or the mortgage rate (real banks earn 100–300 bps over prime on middle-market C&I, but the FRED series for the actual spread, `EEANQ`, was discontinued very tragically in our opinions in 2017).

D Real-Bank Calibration: PNC Financial Services Group (Category III, Q1 2026)

Source filing: The PNC Financial Services Group, Inc., Form 10-Q for the quarter ended March 31, 2026 [11]. We have a "check" for this especially the loan-class totals with the Q1-2026 Financial Supplement [12]. This is a minor note but important for reproducibility: all securities figures are amortized cost (AFS + HTM combined); loans and cash are period-end carrying values. One more note, we pick an 80% "sleeve" of the \$603.0B in total assets. We do this because the remaining 20% (such as consumer revolving and installment loans, goodwill, MSRs, equity investments, and other non-portfolio assets) has no counterpart among our decided ten classes and is excluded; the regulatory scalars (capital, ASF, NCO d) are scaled by 0.80 so the constraint denominators stay consistent with the modeled sleeve of the assets.

D.1 Data Source, Calculation, and Use

Rows marked "—, Constructed" in the source column indicate modeled assumptions rather than values directly quoted from the filing. We tried to limit this, and believe internal banking teams could improve the model with their non-public values subbed in.

Table 3: PNC Financial Services Group Q1-2026 filing sources and model assignments for each of the 10 asset classes.

Source	Calc.	What asset bucket we assign it to
Consolidated Balance Sheet (p. 46): "Cash and due from banks" \$5,646M; "Interest-earning deposits with banks" \$26,053M (note: \$25.3B at FRB)	sum = \$31,699M	bucket 0 cash
Note 3, Table 42 (p. 53): "U.S. Treasury and government agencies" amortized cost, AFS \$26,765M + HTM \$18,880M	= \$45,645M; treated 100% as Treasuries	feeds buckets 1–3
Note 3, Table 45 "Contractual Maturity of Debt Securities" (p. 55): UST & agencies, AFS+HTM now $\leq 1Y$ \$8,282M; 1–5Y \$30,266M; 5–10Y \$4,088M; $> 10Y$ \$3,009M	rule $\leq 5Y$ now short, $> 5Y$ now long: sov_short = \$38,548M, sov_long = \$7,097M	buckets 1 sov_short , 2 sov_long
Note 3, Table 42 (p. 53): "Agency residential mortgage-backed" \$82,070M; "Agency commercial mortgage-backed" \$7,810M	sum = \$89,880M	bucket 4 agency_mbs
Note 3, Table 42 + footnote (e) "Other ... includes state and municipal securities and corporate bonds" \$4,928M	50% now muni = \$2,464M	bucket 5 muni
Note 3, Table 42 (p. 53): "Asset-backed" \$3,636M; "Non-agency RMBS" \$649M; "Non-agency CMBS" \$452M	+ 50% of Other = credit pool \$7,201M	feeds buckets 6–7
Table 9 footnote (a) (p. 9) / Note 3 footnote (b): "Of our total securities portfolio, 97% were rated AAA/AA"	97% of pool goes to corp_aaa = \$6,985M; 3% ergo corp_baa = \$216M	buckets 6 corp_aaa , 7 corp_baa
Table 8 "Loans" (p. 9): "Commercial and industrial" \$221,190M (incl. equipment lease, reclassified 1/1/26); "Commercial real estate" \$34,770M	sum (CRE folded in) = \$255,960M	bucket 8 ci_loan
Table 8 (p. 9): "Residential real estate" \$49,567M	—	bucket 9 mortgage
Table 8 (p. 9): "Total loans" \$360,923M; consumer = home equity \$26,223M + auto \$16,325M + credit card \$7,069M + other consumer \$5,779M	excluded \$55,396M; renormalize over the 10 classes	confirms sleeve = \$482,416M = 80.0% of total assets
Consolidated Balance Sheet (p. 46) / Table 2 (p. 2): "Assets" \$603,028M	this is the underlying basis for the 80% sleeve	scalar total_assets
Table 31 "Basel III Capital" (p. 32): "Total Basel III capital" \$62,102M (CET1 \$47,686M)	sleeve-scaled ($\times 0.80$) now \$49.7B	scalar capital

Source	Calc.	What asset bucket we assign it to
Table 31 (p. 32): “Basel III standardized approach risk-weighted assets” \$472,981M	—	capital-check cross-reference (actual RWA)
Table 11 “Details of Funding Sources” (p. 10): “Total deposits” \$457,648M	input to NCO & ASF	scalars <code>NCO_30d</code> , <code>ASF</code>
Liquidity & Capital Mgmt (p. 27): “uninsured deposits represented approximately 44% of our total deposit base”	$0.44 * \$457,648M = \$201,365M$	input to <code>NCO_30d</code>
Table 11 (p. 10): “Senior debt” \$38,021M; “Subordinated debt” \$4,502M; “FHLB advances” \$21,417M	sum = \$63,940M (term funding)	input to <code>ASF</code>
Consolidated Balance Sheet (p. 46): “Total equity” \$63,676M	—	input to <code>ASF</code>
Liquidity & Capital Mgmt (p. 27): “The LCR for PNC and PNC Bank exceeded the regulatory minimum requirement throughout the first quarter of 2026” (no numeric value); same statement for the NSFR	—	confirms LCR/NSFR > 1.0; tells us NCO/ASF cannot be backed out from disclosure sadly
—, Constructed	assumed: <code>NCO_30d</code> = 30% stressed runoff * uninsured deposits = \$60.4B (*0.80 sleeve scale ergo \$48.3B)	scalar <code>NCO_30d</code> (LCR denominator)
—, Constructed	assumed: <code>ASF</code> = equity + term debt + $0.80 * deposits = \$493.7B * 0.80$ sleeve scale ergo \$395.0B)	scalar <code>ASF</code>
—, Constructed	assumed: “Other” debt split 50/50 muni vs. corporate	<code>muni</code> vs. <code>corp_aaa</code> labeling (Basel-neutral: identical h, ϕ, ω)
—, Constructed	set to 0: TIPS not broken out separately	bucket 3 <code>tips</code>

D.2 Modeling Conventions; Some Notes

We set $\kappa = 13\%$, the effective total-capital minimum PNC actually faces (8% Pillar-1 + 2.5% capital conservation buffer + 2.5% stress capital buffer), rather than the 8% Pillar-1 floor. This brings the capital constraint near binding (RWA ceiling $\approx \$382B$ vs. model RWA $\approx \$301B$).

D.3 Final(!) Calibration

Table 4: Calibrated bank scalars and initial allocation w_0 .

Bank scalar	Value	Asset class	w_0 (%)
Total assets A	\$482.4B	<code>cash</code>	6.57
<code>NCO_30d</code>	\$48.3B	<code>sov_short</code>	7.99
<code>ASF</code>	\$395.0B	<code>sov_long</code>	1.47
<code>capital</code>	\$49.7B	<code>tips</code>	0.00
κ	13%	<code>agency_mbs</code>	18.63
		<code>muni</code>	0.51
		<code>corp_aaa</code>	1.45
		<code>corp_baa</code>	0.04
		<code>ci_loan</code>	53.07
		<code>mortgage</code>	10.27

Checks at this calibration: weights ≥ 0 and sum to 1.000; LCR $(h^\top w_0)A / \text{NCO}_{30d} = 3.35$ (slack, non-binding); NSFR $\text{ASF} / (\phi^\top w_0)A = 1.43$; capital $\kappa(\omega^\top w_0)A = \$39.1B \leq \$49.7B$.

E Q-Q Diagnostics for Fat Tails

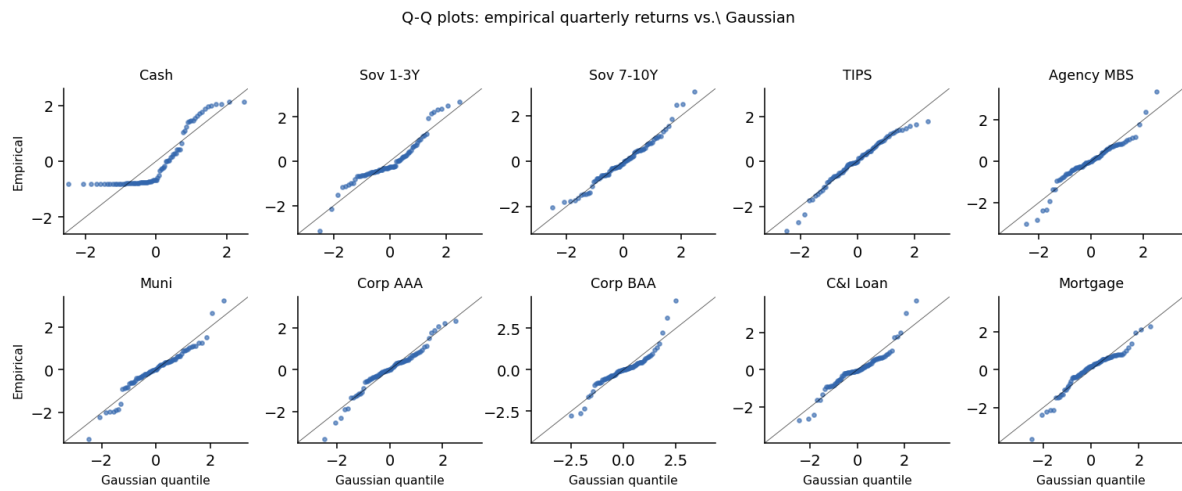


Figure 8: Q-Q plots of empirical quarterly returns against a Gaussian fit, by asset. Deviations from the diagonal in the left tail are the Taleb-style fat-tail signature that motivates the bootstrap risk model.

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